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Problems and prospects of cake filtration theories

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ABSTRACT

The constant vacuum filtration equation for compressible sludge which account for hydrostatic pressure, compressibility coefficient and specific resistance as a variable parameter should be considered further since the equation obtained can only be applicable to a sludge whose Trezaghi's compressibility coefficient is much less than one. This may be the fact that the sludge filtration equation was derived using successive reduction method. Among the three main concepts of sludge filterability, the specific resistance concept is the most popular among the three concepts but most of the equations proposed based on this concept described the specific resistance as a constant average value. This is not in agreement with many research scientists who advocate that the specific resistance parameter should be a local variable one. One of the problems in Carman's equation is the concentration, c which is the mass of dry cake per unit volume of filtrate. This parameter is difficult to evaluate. Also there is a problem of which area to be used in Carman's equation, some previous workers advocate effective area of funnel while another school of thought proposed total area of funnel. The equation proposed which describes specific resistance as a local variable parameter is also limited in use in its present form except the maximum value of the specific resistance is adopted to describe sludge filterability, therefore there's need for derivation of a new sludge filtration equation using another engineering or mathematical method in order to solve the problem of compressibility attribute of sludge called compressibility coefficient or factor, problem of which area to be used and the problem of concentration, c which is the mass of dry cake per unit volume of filtrate.

INTRODUCTION

The treatment of wastewater before disposal is an important step in the minimization of environmental pollution. In the process of treating waste, sludge which is amongst the constituents removed, is by far the highest in volume. The handling and disposal of this sludge is one of the greatest challenges facing the environmental engineers. The sludge has high water content and compressibility attribute and as such it is expedient to dewater it to reduced its volume and prevent environmental health hazard. Sludge is also produced from the treatment of storm water, although it is likely to be less organic in nature compared to waste water sludge (Metcalf and Eddy, 2004).

Mechanical dewatering of sludge is an indispensable step in waste-water treatment. Its importance becomes more pronounced when viewed from the perspective of capital and maintenance costs of filters used in the dewatering process. In recognition of the foregoing, mathematical expressions have been sought over the years to permit a valid description of sludge filtration process (Purchas, 1980). The dewatering of sludge using the constant vacuum filtration method has been adopted in full scale in the 1920s. Since then, a number of equations have been presented by various contributors

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aimed at improving the performance of the sludge filtration process; Carman (1934), Ruth et al. (1933), Coackley and Jones (1956), Heertjes (1964), Anazodo (1974), Ademiluyi (1984, 1985, 1987) and Onosakponome and Onyejekwe (2014) with no agreement regarding which model is better. The dimensional analysis equation presented by Anazodo (1974) which was based on the constant vacuum approach has been criticized because the equation did not account for compressibility attribute of sludge believed to affect sludge filtration process. The problem of compressibility attribute of sludge cake has not been adequately taken care of. It is noteworthy that the search for valid equations to describe sludge dewatering process originated from the fact that the traditional equation proposed by (Carman, 1934, 1938) is only applicable to rigid cakes (Ademiluyi et al., 1983). This is because the laws of Poiseuille and Darcy representing the basis of formulation are only applicable to rigid materials.

Compressibility of cake is one of the reasons why filtration theory is limited. Neither Carman (1934) nor Ruth (1935) gave the qualitative definition of the compressibility attribute used in sludge filtration studies. Ademiluyi et al. (1983)'s investigation, it has been suggested that Carman's concept of compressibility coefficient does not hold for all degree of dilutions. It can only be used for highly diluted sludges when it is in a comparable fluidity state with ordinary water. Besides the above problems, the compressibility attribute of sludge is usually evaluated after a series of filtration experiments based on Carman's theory. Carman's equation has been described to be in error over the years (Heertjes, 1964; Purchas, 1980; Grace, 1953; Anazodo, 1974).

It is therefore reasonable that since the evaluation of compressibility attribute of sludge is based on this theory, compressibility of sludge cakes as propounded by Carman may be also be invalid. It is for this reason that Ademiluyi (1985) derived sludge filtration equation which account for the compressibility attribute of the cake. The compressibility coefficient used in Ademiluyi's equation is that of Terzaghi. Ademiluyi's equation can only be used for sludge whose compressibility attribute which he called compressibility coefficient is less than one.

Sludge dewatering is the separation of solid from the parent slurry. It is an important process finding application in many manufacturing industries and in plants designed for water and waste treatments. Three main concepts have been suggested to evaluate the filterability of sludge over the years. These include specific resistance (Carman, 1934), filtration coefficient (Halff, 1952) and sludge dewaterability number (Ademiluyi, 1987). The concept of specific resistance is the oldest among the three. Carman (1934) pioneered the introduction of the concept of specific resistance for measuring sludge filterability by suggesting the equation:

$$t = \frac{rcvf^2}{2PA^2} + \frac{\mu R V_F}{PA} \quad (1)$$

Where, t, time of filtration (s); r, specific resistance of cake (m/kg); R, resistance of septum; µ, viscosity of filtrate (poise); V_f, volume of filtrate (m^3); p, vacuum pressure (kN/m^2); A, area of filtration (m^2); C, mass of dry cake per unit volume of filtrate (kg/m^2) .

Equation 1 has been modified several times (Anazodo, 1974; Purchas, 1980; Ademiluyi et al., 1982, 1984). Various reasons so far advanced for such modifications have been enumerated and discussed (Ademiluyi, 1986). Ademiluyi (1985) formulated a new equation which describes specific resistance as a local variable parameter. This equation is given as:

$$t = \propto \beta^{sy-(s+2)} \left(\frac{\beta}{s+1} (\varphi^{S+1} - \varphi_o^{S+1}) +^{\beta + Y H_o} \right) e_n \frac{1+\varphi}{1+\varphi} \quad (2)$$

There are problems with Equation 2: the equation can only be used for sludges whose Terzaghi's coefficient of compressibility is found to be less than one (1). The equation was derived based on an assumption that $s < 1$, where s is the coefficient of compressibility. Also Ademiluyi's cake filtration equation has not been solved with any other engineering or mathematical method since 1985, therefore, there's need for the modification of the cake filtration equation to address the problems stated above. Many researchers, (Halff, 1952; Ruth, 1935; White and Gale 1975) have proposed different models aimed at improving the performance of the vacuum filtration process but the exact mathematical formulation governing sludge filtration process is not yet known. Even the commonly used equation as proposed by Carman has been described to be in error (Grace, 1953). Carman's equation was formulated from a combination of Poiseuille's and Darcy's laws. It is therefore applicable to rigid rather than compressible cakes. Also the specific resistance parameter in Carman's equation is usually treated as a constant parameter. This parameter is expected to be a variable since it is a function of cake porosity. The cake porosity decreases from the sludge cake surface to the cake layer closet to the filter septum, there should be a corresponding decrease of specific resistance from the sludge-cake interface to the cake layer closet to the filter septum (Tiller, 1979; Hemant, 1980). The concept of average specific resistance proposed by Ruth has been descried to be inapplicable (Shirato and Tsutomu A. 1972). Also the relationship between the filtrate volume and area of filtration as proposed by Carman has been experimentally found to be invalid (Ademiluyi et al., 1982)

The parabolic relationship between filtrate volume and time of filtration does not hold throughout the filtration cycle (Willis and Tosun, 1980). The variable head and compressibility factor believed to influence sludge dewatering have not been accounted for in Carman's equation (Ademiluyi, 1984). In the light of problems mentioned above, an equation to describe sludge filtration process was derived (Ademiluyi, 1985) but there are still some limitations in his equation. Ademiluyi's theory is not applicable during the early period in cake filtration before the vacuum has assumed a constant value and before enough cake has formed to become the dominant filter resistance. Also Ademiluyi's theory may only be applicable for sludge whose Terzaghi's compressibility coefficient has been found to be less than one as earlier stated in this work. The research in sludge filtration should continue until an acceptable equation which governs sludge filtration phenomenon is derived (Purchas, 1980). Little or no studies have attempted to integrate the aforementioned problems to derive an acceptable equation which governs sludge filtration phenomenon. A large body of research on sludge filtration among the concept of specific resistance, filtration coefficient, sludge dewaterability number, the variable head and compressibility factor, the parabolic relationship between filtrate volume and time of filtration, the average resistance and also the relationship between the filtrate volume and area of filtration models with no agreement regarding which one is better. Many theories have been advanced over the years to describe the important of cake filtration phenomenon as already narrated above. This paper gives a comparative study of such theories with a view to assessing their problems and prospects and consequently will propose a derivation of sludge filtration equation that can combine all or some of the aforementioned problems. An equation of this nature would be useful in the treatment of sludge since it will contain the most important filtration parameters.

FILTRATION THEORIES BASED ON CAKE RESISTANCE

The theory of filtration was pioneered by Almy and Lewis (1912), who filtered chromium hydroxide in a small plate and frame press at a series of constant pressures. They proposed the rate of filtrate and suggested that the equation:

$$\frac{dv}{dt} = \frac{Kp^n}{V^m} \qquad (3)$$

Equation 3 constitutes the basic law of filtration. Where, n and m are indefinite powers of the relationships which the rate of flow $\frac{dv}{dt}$ possesses with pressure, p and volume, v respectively. K is a proportionality constant which varies with type of material to be filtered and the conditions of operation. Also in 1916, Sperry derived another basic filtration of his postulated analogy between the filtration process and groundwater flow. Sperry's theory was based on theoretical rather than experimental considerations. He assumed that since Poiseuille's law holds for groundwater flow, then it should also represent the basic law of filtration. Upon this theoretical base he was able to derive a general equation in which rate of flow was considered to be strictly proportional to the first power of p and v. He suggested a simplified version of Poiseuille's law as:

$$\frac{dv}{dt} = \frac{P}{R} \qquad (4)$$

Where, R is the resistance to the flow of filtrate through the filter cake and the supporting septum. Baker agreed with Sperry's analogy but did not endorse the postulation that the rate of flow was proportional to indefinite power (n and m) of pressure and volume in accordance with the earlier conclusions of Almy and Lewis. On this basis he obtained an integrated equation of the form:

$$V^{(M+1)} = (m+1)KA^2P^nt \qquad (5)$$

Webber and Hershey modified the original equation of Almy and Lewis and gave the integrated form of their modified equation as:

$$\frac{(V)^{2+\theta}}{(A)} = \frac{PL-S}{rii(1+\theta)V}\,\frac{(2+\theta)^{1+\theta}}{(1+\theta t)} \qquad (6)$$

Underwood pointed out that the foregoing equation contains an error and therefore submitted an equation with the introduction of a concept of specific resistance. He suggested an equation of the form.

$$r = r^{11}p^sc\,\frac{(dv)^\theta}{(dt)} \qquad (7)$$

Where, r^{11}, resistance of unit cube when under unit pressure, with unit rate of filtrate flow passing through it; r, average resistance per unit cube of cake.

Since Poiseuille's and Darcy's laws both deal with flow and considering sand bed as a bundle of capillaries, Carman derived the equation of filtration as:

$$t = \frac{\mu rCV^2}{2PA^2} + \frac{\mu RV}{PA} \qquad (8)$$

The above equation gives a straight line when $\frac{t}{V}$ is plotted against V. Carman assumed that although the above equation has its provenance from rigid cake consideration, it could be used for compressible cake. Ruth et al. (1933) played a central role in presenting the idea of specific resistance. Ruth established experimentally that the plot of filtrate volume (v) versus time (t) followed a parabolic relation in line with theoretical predicts based on Carman's equations. He then proposed that the parabolic form of v versus t should be accepted as an axiom on which mathematical relations for filtration might be derived. He also suggested that the specific resistance in Carman's equation should be designated as an average constant value. In 1953, Tiller showed theoretical that the plots of v versus t curves for constant pressure filtration were not perfect parabolas. He said that whenever the pressure drop across the medium was a substantial fraction of the pressure loss across the cake, the average filtration resistance was not constant and that the plot of $\frac{t}{V}$ versus v curve would not be linear, hence the assumptions that the flow rate and independent of distance through the filter bed were found to be invalid. Willis and Tosun (1980) suggested that the deviation from parabolic behavior might be due to the variability of pressure. He also pointed out that such deviations from parabolic behavior are independent of cake compressibility but can be expected if average porosity, pressure difference over only the filter cake and permeability at the cake septum interface are not constant.

MODIFICATION OF CARMAN'S EQUATION

Anazodo (1974) objected to Carman's equation on its formulation point of view. He argued that the approximation of compressible filter cakes to rigid bundles of capillary tubes or to non-compressible sand beds did not make sense. He then developed another basic theoretical equation for the filtration compressible sludge using FMLLxLyLz dimensional analyses to arrive at the equation:

$$V = \left(\frac{A^2}{Cr}\right)\frac{A^{1/2}\, P\, C^{1/2} r^{1/2}\, t}{\mu} \quad f \tag{9}$$

Since the relationship between v and t has been established to be parabolic, Anazodo substituted f = ½ to obtain a dimensional equation for sludge filtration as

$$V^2 = \frac{P A^{5/2}\, t}{\mu C^{1/2} r^{1/2}} \tag{10}$$

The derivation of the dimensional equation was not acceptable to white Gale who advocated that:

- Carman's equation which only predicts volume of filtrate to be just proportional to the area was preferable.
- Anazodo did not justify the prediction that the volume of filtrate obtained after a fixed time is proportional to the filtration area to the power of $5/4$.
- Anazodo need not assume f = ½. Gale (1975) advocated that Anazodo's partial equation should be written as:

$$V^2 = P\mu^{-1} A^{2b} t\, (Cr) 2b - 3, \tag{11}$$

So that the relationship between V and A should be experimentally determined.

If b = 1, Carman's equation is implied and If b = $5/4$, Anazodo's equation is arrived at. A consensus was reached by both parties that the determination of the correct value of 'b' based on theoretical or experimental considerations would guide the choice of filtration equation.

Ademiluyi et al. (1982) conducted an investigation to determine experimentally the values of the exponent 'b' which relates the volume of filtrate to the area of filtration. The average value of 'b' was found to be 0.91±0.02, if calculations were made with the total area of filtration while the value of b was 1.38 ± 0.02, if the effective area of filtration was used in the analysis of experimental data. They suggested that the total area of filtration should be used in Carman's equation while the effective area of filtration should be used in the dimensional equation for sludge filtration at constant pressure. The equations suggested by Ademiluyi after the substitution of 'b' in the partial dimension equation are:

$$V^2 = \frac{P A_t\, 1.82t}{\mu (Cr)^{1.18}} \tag{12}$$

$$V^2 = \frac{P A_{eff}^{2.76t}}{\mu (Cr)^{0.24}} \tag{13}$$

Hemant (1980) modified Ruth's theory by empirically incorporating particle migration in the cake in case of dilute slurries. Hemant's modified equation is given as:

$$\log V = \frac{1}{2+a} \log ft + \frac{1}{2+a} \frac{\log(2+a)p}{m_{av}^{\propto} C} \tag{14}$$

Commenting on the variability of specific resistance, Hermant claimed that the assumption that average specific resistance is constant, is not valid for analysis of data collected only about 20 min. In view of the various limitations of the traditional equation, a new equation was formulated which describes specific resistance as a local variable parameter (Ademiluyi, 1985). This equation is given as:

$$t = \propto \beta^{sy-(s+2)} \left(\frac{\beta}{s+1} (\varphi^{S+1} - \varphi_o^{S+1}) + ^{\beta + Y\, H_o} \right)$$
$$e_n \frac{1+\varphi}{1+\varphi} \tag{15}$$

It is noteworthy that Equation 15 takes accounts for the hydrostatic head and compressibility attribute of the sludge cake which are believed to have influence on sludge dewatering but the equation is only applicable to sludges whose compressibility coefficient is less than one.

THEORY BASED ON FILTRATION COEFFICIENT

Finding poor correlation between theoretical and practical results on rotary vacuum filters, using the equations of Ruth, Halff (1952) developed an approval based upon Terzaghi's theoretical treatment of the consolidation of soils. He proposed the hypothesis that the driving force varies with time and distance. In this respect it is different from filtration equation previously developed by Carman and Ruth. The derivation of Halff's equation established the fact that filtration follows the parabolic relationship until the cake reaches 80% filtration or consolidation. He proposed that:

$$C_{fi} = \frac{SYi}{\mu CoS} \tag{16}$$

THEORY BASED ON THE CONCEPT OF SLUDGE DEWATERABILITY NUMBER (SDN)

Even though the specific resistance parameter is a good measure of sludge filterability, it does not have consistent unit. This inherent problem in the concept leads to loss of bench marks. In the light of the foregoing, a dimensionless number referred to as the sludge dewaterability number was proposed (Ademiluyi, 1987). Experiments showed that, SDN is a very good measure of sludge filterability. It is given as:

$$SDN = \frac{DH(C_O - C_f)}{C_C\, V_i\, t} + \frac{Ho}{V_i\, t} \tag{17}$$

DISCUSSION

Many sludge filtration equations based on different concepts have been proposed since the works of Carman and Ademiluyi. Most of the equations proposed are based on the concept of filtration resistance and compressibility attribute of sludge. Equation 15 which is based on the concept of compressibility attribute of sludge may be considered further since it generally entails an assumption of compressibility coefficient which is much more less than one in the two sludges tested. The modified forms of Carman's equation including the dimensional equations cannot be accepted since the specific resistance parameters in these equations has been described as constant in the equations. Shirato and Tsutomu A. (1972) has already stated that the specific resistance parameter should not been taking as an average value since cake porosity decreases from sludge cake interface towards the cake layer closest to the septum. Equation 15 takes into account all the main parameters believed to influence sludge dewatering. Such factors as hydrostatic head and compressibility attributes of sludge which are not accounted for in the previous equations are incorporated into this expression. Such incorporation makes the equation more complex than the traditional and dimensional equations. This is the most apparent limitation of this equation; the equation cannot be used for sludge whose compressibility factor is more than one. Another limitation of Equation 15 is that the equation cannot be used to quantify sludge dewaterability in its present form since specific resistance in the equation is presented as a variable rather than a constant value. There's also difficulty in finding out whether the expression is dimensionally homogenous like the Carman's and Anazodo's equations. This is because some variables in the equation have exponent's' which is not a pure dimensionless number and whose value needs to be determined in each experimental investigation. There is also a problem of which area to be used in Carman's equation, some previous workers advocate effective area of funnel while another school of thought proposed total area. Also in Carman's equation, there is a problem of the concentration, c which is the mass of dry cake per unit volume of filtrate, this parameter is very difficult to evaluate.

CONCLUSION

The constant vacuum filtration equation for compressible sludges should be considered further since the equation obtained can only be applicable for sludges whose Trezaghi's compressibility coefficient is much less than one. This may be the fact that the sludge filtration equation was derived using successive reduction method. Among the three main concepts of sludge filterability, the specific resistance concept is the most popular among the three concepts but most of the equations proposed based on this concept described the specific resistance as a constant average value. This is not in agreement with many research scientists who advocate that the specific resistance parameter should be a local variable one. The equation proposed which describes specific resistance as a local variable parameter is also limited in use in its present form except the maximum value of the specific resistance is adopted to describe sludge filterability, therefore there's need for a new sludge filtration equation to solve all or some of the problems narrated above.

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Appendix

List of symbols

A = Area of filtration (m^2)
A_{eff} = Effective area of funnel
C_o = Initial concentration of sludge
C_c = Mass of dry cakes deposited per unit volume of filtrate (kg/m^2)
C_f = Concentration of filtrate
C_{fi} = Filtration coefficient
ΔH = Change in headloss
r = Specific resistance
g = Acceleration due to gravity (m/s^2)
H_o = Initial driving head (m)
H = Driving head at any time t, (m)
h_m = mercury rise in manometer (m)
h_f = Head loss (m)
L = Thickness of the cake (m)
L_m = Thickness of the medium (m)
P = Applied vacuum pressure (KN/m^2)
P_L = Hydraulic pressure (N/m^2)
P_s = Compressive drag pressure of solids (N/m^2)
q or dv/dt = Flow rate (m^3/s)
R^* = Specific resistance at any arbitrary head loss (s^2/g)
R = Average specific resistance (m/kg)
R^1 = Cake resistance (m^{-2})
R_1 = Local flow resistance (m/kg)
R_m = Medium or septum resistance (m^{-2})
s = Compressibility factor (cm^2/g)
SDN = Sludge dewaterability number (dimensionless)
S_c = compressibility coefficient as defined by Carman.
S_o = Initial solid content of sludge (kg/m^3)
t = Time taken to obtain filtrate (s)
v = Volume of filtrate (m^3)
V_i = instantenous velocity
a = An arbitrary constant
$\propto_{av}$ = Average specifiv resistance
W = Mass of dry solids deposited per unit area (kg/m^2)
β = Vacuum pressure (KN/m^2)
p_f = Density of filtrate (kg/m^3)
p_m = Density of mercury (kg/m^3)
γ = Specific weight of filtrate (N/m^3)
μ = Viscosity of filtrate (poise)
Ψ_o = Hydrostatic pressure due to filtrate per unit applied vacuum pressure at t = 0
Ψ = Hydrostatic pressure due to filtrate per unit applied vacuum pressure at any time 't'
Δ = Constant of proportionality (dimensionless)
γ_L = Specific weight of liquid
θ = Coefficient of scouring effect
C_c = Concentration of cake